

Yang-Mills condensate dark energy coupled with matter and radiation

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Abstract

The coincidence problem is studied for the dark energy model of effective Yang-Mills condensate (YMC) in a flat expanding universe during the matter-dominated stage. The YMC energy $\rho_y(t)$ is taken to represent the dark energy, which is coupled either with the matter $\rho_m(t)$, or with both the matter and the radiation components $\rho_r(t)$. The effective YM Lagrangian is completely determined by quantum field theory up to 1-loop order with an energy scale $\kappa^{1/2} \sim 10^{-3}\text{eV}$ as a model parameter, and for each coupling, there is an extra model parameter introduced. Beyond the non-coupling case, we have extensively studied four types of coupling models: 1) the YMC decaying into the matter at a rate $\Gamma \sim 0.5H_0$, where H_0 is the Hubble constant; 2) the matter decaying into the YMC at a rate $\Gamma \sim 0.02H_0$; 3) the YMC decaying into both the matter (at $\Gamma \sim 0.5H_0$) and the radiation (at $\Gamma' \sim 1.8 \times 10^{-4}H_0$); 4) both the matter (at $\Gamma \sim 0.5H_0$) and the radiation (at $\Gamma' \sim 1.8 \times 10^{-4}H_0$) decaying into the YMC. In each of these four models, we have also explored various couplings. For all these models and for a variety of functional forms of the couplings Γ and Γ' , it is found that the overall feature of the cosmic evolution for the YMC component is similar. Starting from the equality of radiation-matter $\rho_{mi} = \rho_{ri}$, for generic initial conditions of ρ_{yi} subdominant by a factor ranging over 8 orders of magnitude from 10^{-10} to 10^{-2} , the models always have a scaling solution during the early stages, and the YMC always levels off at late time and becomes dominant, so that the universe transits from the matter-dominated into the dark energy-dominated stage at $z \simeq (0.3 \sim 0.5)$, and evolves to the present state with $\Omega_y \simeq 0.7$, $\Omega_m \simeq 0.3$, and $\Omega_r \simeq 10^{-5}$. For the matter and radiation components, their evolutions depend on how they couple to the YMC. It is found that, if the YMC decays into a component with a rate being a constant or depending on ρ_y , as in Model 1 and Model 3, then in the early stages one has $\rho_m(t) \propto a(t)^{-3}$ and $\rho_r(t) \propto a(t)^{-4}$ like the non-coupling case, but later around $z \sim 0$, this component also stops decreasing and levels off, asymptotically approaching to a constant value, just as $\rho_y(t)$ does. In these cases, the equation of state (EoS) of the YMC $w_y = \rho_y/p_y$ crosses over -1 around $z \sim 2$ and takes the current value $w_y \simeq -1.1$ at $z = 0$, consistent with the recent preliminary observations on supernovae Ia. But if the coupling is such that the matter decays into the YMC in Model 2

or if both the matter and radiation decay into the YMC in Model 4, then $\rho_m(t) \propto a(t)^{-3}$ and $\rho_r(t) \propto a(t)^{-4}$ approximately for all the time, and w_y approaches to -1 but does not cross over -1 . We have also demonstrated explicitly that, the coupled dynamics for $(\rho_y(t), \rho_m(t))$ in Model 1, or for $(\rho_y(t), \rho_m(t), \rho_r(t))$ in Model 3, as $t \rightarrow \infty$, is a stable attractor; in Model 2 and Model 4 the dynamics of $\rho_y(t)$ has a stable attractor as $t \rightarrow \infty$. Therefore, under very generic circumstances, the existence of the scaling solution during the early stages and the subsequential exit from the scaling regime around $z \simeq (0.3 \sim 0.5)$ are inevitable. Thus the coincidence problem is naturally solved in the effective YMC dark energy models.

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1. Introduction

The observations on the cosmic microwave background radiation (CMB)[1] suggests a flat universe. This observation together with that from the Type Ia Supernova (SN Ia) [2] implies that the universe consists of some mysteries dark energy ($\Omega_\Lambda \sim 0.73$), dark matter ($\Omega_d \sim 0.23$), and ordinary baryon matter ($\Omega_b \sim 0.04$), and a radiation component $\Omega_r \sim 10^{-5}$. This is also supported by and the large scale structure of the universe [3]. The dark energy as the dominant cosmic energy component drives the current accelerating expansion of the universe. The simplest model for the dark energy is the cosmological constant Λ , which corresponds to a homogeneous and time-independent energy density $\rho_\Lambda \sim 5.8h_0^2 \times 10^{-11}\text{ev}^4$ with an EoS of $w_\Lambda = p_\Lambda/\rho_\Lambda = -1$. Although this model can account for the observation so far, it has a coincidence problem. The observations show that the present value of the energy density for the matter component $\rho_m = \rho_d + \rho_b$ is about one third of ρ_Λ , but it varies with time as $\rho_m(t) \propto a(t)^{-3}$. So, for example, at an earlier time of radiation-matter equality with redshift $z \simeq 3454$ [4], ρ_Λ should be a very fine tuned value $\simeq 6.3 \times 10^{-11}\rho_m(t)$. Otherwise, a slightly variant initial value of ρ_Λ would lead to a value of the ratio ρ_Λ/ρ_m drastically different from the observed one. This is called the coincidence problem.

One class of models aiming at solving the coincidence problem is based upon the dynamics of some scalar field ϕ , such as quintessence [5], K-essence [6], tachyon [7], phantom [8], and quintom [9], etc. These models can give rise to certain desired features of evolutionary dynamics, such as scaling solutions [10] and tracking behavior. As a common point, these scalar models need to make use of some special forms of the potential $V(\phi)$ with certain chosen parameters. For example, in the quintessence model, one needs to choose $V(\phi) \propto (M/\phi)^\alpha$ with α and M being a positive number, or $\propto e^{-\phi/m_{pl}}$ [11]. In phantom model one may take $V(\phi) = V_0[\cosh(\alpha\phi/m_{pl})]^{-1}$ [12]. Moreover, phantom models typically introduce a negative kinetic energy term $-\dot{\phi}^2/2$. Some of these models are expected to be low energy effective field theory, coming from some fundamental field theory, others are simply introduced by hand.

As far as cosmological observations are concerned, a cosmological model has to give the current status: $\Omega_\Lambda \sim 0.7$ and $\Omega_m \sim 0.3$. Besides, it is safer for a model not to contradict the conventional scenario in the Big Bang model from the energy scale $\sim 1\text{Mev}$ down to the present. Therefore, one idea to solve the coincidence problem is that during the early stages of the expansion the dark energy density need not to be a constant, but varies with time. Even its EoS w need not to be close to -1 in early stages. Only at some rather recent moment has it become dominant and acquired an EoS $w \sim -1$. In order to allow the conventional cosmological processes, such as the nucleosynthesis and the recombination, etc., to have been occurred in the past, the dark energy component should be subdominant to the matter component early stages.

In the approach of the scalar field models, certain coupling has been introduced between the scalar field dark energy and the matter [13] [14]. With some particular choice of the model parameters, there can exist a scaling solution of dynamics, in which the dark energy density is proportional to that of matter during the early stages of evolution. However, to achieve the tracking solution, some scalar models need to have a very large coupling, so that the universe would enter the acceleration stage soon after the matter era. This would result in a picture of structure formation, totally different from that required by the observations. To remedy this defect some models [15] [16] introduce certain particular form of couplings, but still the entrance to the accelerating stage is a little earlier than what observations suggested. On the other hand, it has been pointed

out that the k-essence models always have, at some stage, the difficulty of superluminal propagation, leading to violation of causality, and the EoS and the sound speed of k-essence could be greater than > 1 [17]. Besides the unconventional negative kinetic energy, after the EoS w cross -1 , a class of scalar models may suffer from severe quantum instabilities and the Big Rip singularity, i.e. either the energy density ρ , or the pressure p , may grow to infinity within a finite time. More recently, an overall estimate of scalar models has been given on the issue of coincidence problem. By examining the most general form of scalar Lagrangian with a generic coupling between scalar dark energy and dark matter, it has been shown that a vast class of scalar models, including all the models in current literature, have the difficulty to implement both a scaling solution without singularity and a sequence of expansion epochs that required by standard cosmology, such as the radiation-dominated, matter-dominated, and dark energy-dominated epochs [18]. Therefore, the coincidence problem still remains after over a decade of extensive studies [19].

The introduction of the quantum effective YMC into cosmology [20] has been motivated by the fact that the $SU(3)$ YMC has given a phenomenological description of the vacuum within hadrons confining quarks, and yet at the same time all the important properties of a proper quantum field are kept, such as the Lorentz invariance, the gauge symmetry, and the correct trace anomaly [21]. Quarks inside a hadron would experience the existence of the Bag constant, B , which is equivalent to an energy density $\rho = B$ and a pressure $p = -B$. So quarks would feel an energy-momentum tensor of the vacuum as $T_{\mu\nu} = B \text{diag}(1, -1, -1, -1)$. This non-trivial vacuum has been formed mainly by the contributions from the quantum effective YMC, and from the possible interactions with quarks. Our thinking has been that, like the vacuum of QCD inside a hadron, what if the vacuum of the universe as a whole is also filled with some kind of YMC. Gauge fields play a very important role in, and are the indispensable cornerstone to, particle physics. All known fundamental interactions between particles are mediated through gauge bosons. Generally speaking, as a gauge field, the YMC under consideration may have interactions with other species of particles in the universe. In our previous studies of [23, 24], the possible interaction of the YMC with other cosmic components have not been examined. However, unlike those well known interactions in QED, QCD, and the electro-weak unification, here at the moment we do not yet have a model for the details of the microscopic interactions between the YMC and other particle. Therefore, in this paper on the dark energy model, we will adopt a simple description of the possible interactions between the YMC and other cosmic particles. That is, we introduce coupling terms in the continuity equations of cosmic energy densities, such as the YMC, the matter and the radiation, study the cosmic evolution of the universe from the matter dominated era up to the present. As shall be seen, in our model the current status of the universe turns out to be a natural result of evolutionary dynamics driven by the effective YMC as the dark energy, plus the matter and the radiation that are coupled to the former. What is important is that this has been achieved with a choice of the initial value of the fractional energy density of the YMC ranging from 10^{-10} to 10^{-2} . As a novel feature in contrast to the non-coupling models, the coupling YMC dark energy models can give rise to an EoS of dark energy w_y crossing -1 , say $w_y \sim -1.1$ at present. The coincidence problem can be solved.

In section 2, as a basis for the setup, an introduction is given to the effective Yang-Mills condensate theory, and the dynamic equations for the three components in the Robertson-Walker spacetime are derived. Section 3 is about the simplest case of non-coupling. Section 4 studies the dynamic cosmic evolution with

the effective YMC decaying into the matter component. There are two dynamic equations for the YMC and the matter, respectively. Section 5 studies the model in which the matter decays into the YMC. The matter component has different dynamic evolutions especially at late stages for these two coupling models. Nevertheless, in both these two models, the YMC has a scaling solution and a tracking behavior as a natural outcome of the dynamic evolution. Section 6 extends to the general case with the YMC coupling to both the matter and the radiation. Now one has one more coupled dynamic equation and an extra coupling for the radiation component. Two cases are studied, the YMC decaying into the matter and radiation, and the matter and radiation decaying into the YMC. The dynamics is examined parallel to sections 4 and 5. In each of these four models, various functional forms of coupling have been explored. The major part of the study is in sections 4, 5, and 6, which contain the calculations and the results. Section 7 gives an analysis on asymptotic behavior of the dynamic system at $t \rightarrow \infty$. It is found that there exists a unique attractor in the asymptotic region, which is stable against perturbations. Section 8 contains a summary and discussions.

Throughout this paper we will work with unit, in which $c = \hbar = k_B = 1$.

2. YM condensate as dark energy

In the effective YMC dark energy model, the effective YM field Lagrangian is given by [20] [21]:

$$L_{eff} = \frac{1}{2}bF(\ln|\frac{F}{\kappa^2}| - 1) \quad (1)$$

where κ is the renormalization scale of dimension of squared mass, $F \equiv -\frac{1}{2}F_{\mu\nu}^a F^{a\mu\nu} = E^2 - B^2$ plays the role of the order parameter of the YMC. In this paper, for simplicity, we only discuss the pure ‘electric’ case, $F = E^2$. The Callan-Symanzik coefficient $b = (11N - 2N_f)/24\pi^2$ for $SU(N)$ with N_f being the number of quark flavors. For the gauge group $SU(2)$ considered in this paper, one has $b = 2 \cdot 11/24\pi^2$ when the fermion’s contribution is neglected, and $b = 2 \cdot 5/24\pi^2$ when the number of quark flavors is taken to be $N_f = 6$. For the case of $SU(3)$ the effective Lagrangian in Eq.(1) leads to a phenomenological description of the asymptotic freedom for the quarks inside hadrons [21]. It should be noticed that the $SU(2)$ YM field is introduced here as a model for the cosmic dark energy, it may not be directly identified as the QCD gluon fields, nor the weak-electromagnetic unification gauge fields, such as Z^0 and W^\pm . As will be seen later, the YMC has an energy scale characterized by the parameter $\kappa^{1/2} \sim 10^{-3}$ ev, much smaller than that of QCD and of the weak-electromagnetic unification. An explanation can be given for the form in Eq.(1) as an effective Lagrangian up to 1-loop quantum correction [21]. A classical $SU(N)$ YM field Lagrangian is

$$L = \frac{1}{2g_0^2}F,$$

where g_0 is the bare coupling constant. As is known, when the 1-loop quantum corrections are included, the bare coupling constant g_0 will be replaced by the running coupling g as the following [21][22]

$$g_0^2 \rightarrow g^2 = \frac{4 \cdot 12\pi^2}{11N \ln(\frac{k^2}{k_0^2})} = \frac{2}{b \ln(\frac{k^2}{k_0^2})},$$

where k is the momentum transfer and k_0 is the energy scale. To build up an effective theory [21], one may just replace the momentum transfer k^2 by the field strength F in the following manner:

$$\ln(\frac{k^2}{k_0^2}) \rightarrow 2 \ln|\frac{F}{\kappa^2}| = 2(\ln|\frac{F}{\kappa^2}| - 1),$$

yielding Eq.(1). We like to point out that the renormalization scale κ is the only parameter of this effective YM model, and its value should be determined by comparing the observations. In contrast to the scalar-field dark energy models, the YMC Lagrangian is completely fixed by quantum corrections up to order of 1-loops, and there is no room for adjusting its functional form. This is an attractive feature of the effective YMC dark energy model.

From Eq.(1) we can derive the energy density and the pressure of the condensate [20, 23] in the flat R-W spacetime:

$$\rho_y = \frac{1}{2}\epsilon E^2 + \frac{1}{2}bE^2, \quad (2)$$

$$p_y = \frac{1}{6}\epsilon E^2 - \frac{1}{2}bE^2, \quad (3)$$

where ϵ is called the dielectric constant of the YMC, given by [21]

$$\epsilon = 2 \frac{\partial L_{eff}}{\partial F} = b \ln \left| \frac{F}{\kappa^2} \right|. \quad (4)$$

The EoS of YMC is given by

$$w_y = \frac{p_y}{\rho_y} = \frac{y-3}{3y+3}, \quad (5)$$

where

$$y \equiv \frac{\epsilon}{b} = \ln \left| \frac{E^2}{\kappa^2} \right| \quad (6)$$

is a dimensionless quantity, in terms of which the energy density and pressure of the YMC will be given by

$$\rho_y = \frac{1}{2}b\kappa^2(y+1)e^y, \quad (7)$$

$$p_y = \frac{1}{2}b\kappa^2\left(\frac{1}{3}y-1\right)e^y. \quad (8)$$

One sees that, to ensure that the energy density be positive in any physically viable model, the allowance for the quantity y should be $y > -1$, i.e. $F > \kappa^2/e \simeq 0.368\kappa^2$. Before setting up a cosmological model, the EoS w_y itself as a function of F is interesting. From Eqs.(2) and (3) one sees that the YMC exhibits an EoS of radiation with $p_y = \frac{1}{3}\rho_y$ and $w = 1/3$ for a large dielectric $\epsilon \gg b$ (i.e. $F \gg \kappa^2$). On the other hand, for $\epsilon = 0$ (i.e. $F = \kappa^2$), which is called the critical point, the YMC has an EoS of the cosmological constant with $p_y = -\rho_y$ and $w = -1$. The latter case occurs when the YMC energy density takes on the value of the critical energy density $\rho_y = \frac{1}{2}b\kappa^2$ [20]. It is this interesting property of the EoS of YMC, going from $w = 1/3$ at higher energies ($F \gg \kappa^2$) to $w = -1$ at low energies ($F = \kappa^2$), that makes it possible for the scaling solution [10] for the dark energy component to exist in our model. More interestingly, this transition is smooth since w is smooth function of y in the range $(-1, \infty)$. Now we ask the question: Can w_y cross over -1 ? By looking at Eq.(5) for w_y , we see that w_y only depends on the value of the condensate strength F . In principle, $w_y < -1$ can be achieved as soon as $F < \kappa^2$. Moreover, in regards to the behavior of w_y as a function of F , this crossing is also smooth. However, as shall be shown explicitly later, when the YMC is put into a cosmological model as the dark energy component, together with the radiation and matter components, to drive the expansion of the universe, the value of F can not be arbitrary, it comes out as a function of time t and has to be determined by the dynamic evolution. Specifically, when the YMC does not decay into the matter and radiation, w_y can only approaches to -1 asymptotically, but will not cross

over -1 . On the other hand, when the YMC decays into the matter and/or radiation, w_y does cross over -1 , and, depending on the strength of the coupling, w_y will settle down to an asymptotic value ~ -1.17 . As a merit, in this lower region of $w_y < -1$, all the physical quantities ρ_y , p_y , and w_y behave smoothly, there is no finite-time singularities that are suffered by a class of scalar models.

Now we put the YMC in to the cosmic setting, which is assumed to be a spatially flat ($k = 0$) Robertson-Walker spacetime

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j. \quad (9)$$

As it stands, the present universe is filled with three kinds of major energy components, the dark energy, the matter, including both baryons and dark matter, and the radiation. In our model, the dark energy component is represented by the YMC, and the matter component is simply described by a non-relativistic dust with negligible pressure, and the radiation component consists of CMB and possibly other particles, such as neutrinos, if they are massless. Since the universe is assumed to be flat, the sum of the fraction densities is $\Omega = \Omega_y + \Omega_m + \Omega_r = 1$, where the fractional energy densities are $\Omega_y = \rho_y/\rho$, $\Omega_m = \rho_m/\rho$, and $\Omega_r = \rho_r/\rho$. The overall expansion of the universe is determined by the Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_y + \rho_m + \rho_r), \quad (10)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_y + 3p_y + \rho_m + \rho_r + 3p_r), \quad (11)$$

in which all these three components of energy contribute to the source on the right-hand side of the equations. The dynamical evolutions of the three components are determined by their equations of motion, which can be written as equations of conservation of energy [20, 23]:

$$\dot{\rho}_y + 3\frac{\dot{a}}{a}(\rho_y + p_y) = -Q_m - Q_r, \quad (12)$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m = Q_m, \quad (13)$$

$$\dot{\rho}_r + 3\frac{\dot{a}}{a}(\rho_r + p_r) = Q_r, \quad (14)$$

where Q_m represents the energy exchange between the YMC and the matter, and Q_r between the YMC and the radiation, respectively. In the natural unit, both quantities have the dimension of $[energy]^5$. The couplings Q_m and Q_r are phenomenological, and their specific forms will be addressed later. The sum of Eqs. (12), (13), and (14) guarantees that the total energy is still conserved. As is known, Eq.(11) is not independent and can be derived from Eqs.(10), (12), (13), and (14). It is noted that once the couplings Q_m and Q_r are introduced as above, they will bring two new parameters in our model. When $Q_m > 0$, the YMC transfers energy into the matter, and this could be implemented, for instance, by the processes with the YMC decaying into pairs of matter particles. On the other hand, when $Q_m < 0$, the matter transfers energy into the YM condensate. Similarly, when $Q_r > 0$, the YMC transfers energy into the radiation. Therefore, in the most general case of coupling, there will be three model parameters: Q_m , Q_r , and κ .

In the following computations, it is simpler to employ the following functions rescaled by the critical energy density $\frac{1}{2}b\kappa^2$ of YMC,

$$x \equiv \frac{\rho_m}{\frac{1}{2}b\kappa^2}, \quad (15)$$

$$r \equiv \frac{\rho_r}{\frac{1}{2}b\kappa^2}, \quad (16)$$

$$q_m \equiv \frac{Q_m}{\frac{1}{2}b\kappa^2}, \quad (17)$$

$$q_r \equiv \frac{Q_r}{\frac{1}{2}b\kappa^2}. \quad (18)$$

Here the dimensionless functions x and r are simply the rescaled energy density of the matter and radiation, respectively, and the rescaled exchange rates q_m and q_r have unit of $[time]^{-1}$. Then, in terms of x , y , and r , the dynamical evolutions given in Eqs. (10), (12)-(14) can be recast into:

$$\frac{dy}{dN} + \frac{4y}{2+y} = -\frac{q_m + q_r}{Hh(2+y)e^y}, \quad (19)$$

$$\frac{dx}{dN} + 3x = \frac{q_m}{Hh}, \quad (20)$$

$$\frac{dr}{dN} + 4r = \frac{q_r}{Hh}, \quad (21)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 h^2, \quad (22)$$

where the variable $N = \ln a(t)$, the function $h = \sqrt{(1+y)e^y + x + r}$, and the constant $H = \sqrt{4\pi G b \kappa^2 / 3}$. Note that H is not exactly the present Hubble constant H_0 . From Eq.(22) one can see that it is the quantity Hh that determines the actual expansion rate of the universe, and the present value of Hh is identified as the Hubble constant H_0 . However, as our calculations will show later, the value of $h \sim 1.07$ at present, so approximately $H \simeq H_0$. It should be emphasized that H is not an independent parameter, it is fixed by the model parameter κ . Once the YMC is put into the cosmological context, one can estimate the order of magnitude of κ as follows. The critical density $\rho_c = 8.099h_0^2 \times 10^{-11} \text{ev}^4$ with the Hubble parameter $h_0 \simeq 0.72$, and the current value of the dark energy density should be $\rho_y = \Omega_y \rho_c \simeq 0.7\rho_c$. As calculations will show, the present value of the factor $(1+y)e^y \simeq 0.8$ in Eq.(7), and $\rho_y \simeq 0.8 \times \frac{1}{2}b\kappa^2$. So one has

$$\kappa^{1/2} \simeq 5 \times 10^{-3} h_0 \text{ ev}. \quad (23)$$

This energy scale is much smaller than those typical energy scales occurring in the standard model of particle physics, such as $\sim 10^2$ Mev for QCD, and $\sim 10^2$ Gev for the weak-electromagnetic unification. Therefore, for lack of an explanation of the origin of this energy scale κ within the standard model, we may have to regard the YM condensate as a new physics beyond the standard model of particle physics. In this sense, like other dark energy models, the fine-tuning problem, i.e. why κ has just this small value, also exists in our model.

The set of equations (19) – (22) hold for a generic stage of cosmic expansion driven by the combination of the radiation, the matter, and the dark energy. In this paper we focus on the matter-dominated era and the subsequent accelerating era. In particular, we like to see how the cosmic expansion evolves and transits from the matter-dominated to the accelerating era. In the remaining of the paper, we always take the initial condition to be at the time t_i of the equality of radiation-matter (with a redshift $z = 3454_{-392}^{+385}$ [4])

$$\Omega_{mi} = \Omega_{ri}, \quad (24)$$

with the subscript i denoting the initial value. Of course, once the initial value Ω_{yi} for the YMC is given, one has immediately

$$\Omega_{mi} = \Omega_{ri} = \frac{1}{2} - \frac{1}{2}\Omega_{yi}. \quad (25)$$

In order to keep the main features of the conventional scenario of cosmic expansion, it is also assumed that initially the matter and the radiation are dominant, $\Omega_{mi} = \Omega_{ri} \simeq 1/2$, and the YMC is subdominant

$$\Omega_{yi} \ll 1/2. \quad (26)$$

In Ref.[24] we considered the constraints on the YMC energy density at an earlier stage. There the initial condition was taken at a redshift $z \simeq 10^{10}$, corresponding to an energy ~ 1 Mev, during the radiation stage, when the Big Bang nucleosynthesis processes took place. The upper bound has been found to be $\Omega_y \leq 0.26\Omega_r$ at $z \simeq 10^{10}$. Afterwards up to $z \sim 3000$, both ρ_r and ρ_y evolved approximately in a similar way $\propto a(t)^{-3}$. Thus, at the equality of radiation-matter with $z \simeq 3500$, this consideration will give an upper bound $\Omega_{yi} \sim 0.1$. Within this restriction, the initial value Ω_{yi} may still be allowed to vary in a very broad range. In the following we take a safe value

$$\Omega_{yi} \leq 10^{-2} \quad (27)$$

as the upper bound both for illustration purpose. This choice has been made in concordance with the thinking that the dark energy component has been existing in the universe from the equality of radiation-matter, but its initial relative contribution is bounded by a few percent of the total, so that during most of the history of the universe the cosmic evolution will be the same as in the standard Big Bang model. Only quite recently has the dark energy component become dominant and modified the cosmic evolution considerably.

3. Non-coupling Case

The simplest case is the non-coupling with $Q_m = 0$ and $Q_r = 0$ in Eqs.(12), (13), and (14). Then there is only one model parameter κ that has already been fixed in Eq.(23). Each component evolves independently in the expanding RW spacetime. To solve the dynamic equations, we take the initial YMC at the time t_i to be in the broad range

$$\Omega_{yi} = (10^{-10}, 10^{-2}), \quad (28)$$

consistent with the restriction (27), that is, the initial value of ρ_{yi} ranges over eight orders of magnitude. In terms of y , x , and r , the initial condition above can be written as

$$y_i = (1, 16.12), \quad (29)$$

$$x_i = r_i = 1.7 \times 10^{10}. \quad (30)$$

The equations (19) (20) (21) with $q_m = q_r = 0$ are solved easily for $\rho_y(t)$, $\rho_m(t)$, and $\rho_r(t)$, which are shown as a function of redshift z in Fig.1. Both $\rho_m(t) \propto a^{-3}(t)$ and $\rho_r(t) \propto a^{-4}(t)$ decrease monotonically at their fixed slope, respectively, and do not level off as $t \rightarrow \infty$. More interesting is the evolution of YMC energy density. In the early stage $\rho_y(t)$ is subdominant to $\rho_m(t)$ and $\rho_r(t)$, and decreases at a slope between those of $\rho_r(t)$ and $\rho_m(t)$, tracking the matter. Later, $\rho_y(t)$ gradually levels off and approaches to a constant.

At $z \sim 0.35$, $\rho_y(t)$ starts to dominate over $\rho_m(t)$ and the accelerating expansion takes over. This exit from the subdominant region (scaling) to the dominant region is naturally realized. It is important to notice that, as long as the initial value is in the broad range of Eq.(28), $\rho_y(t)$ always has the same asymptotic value as $t \rightarrow \infty$. By the way, the first order differential equations (20) and (21) for $x(t)$ and $r(t)$ have no fixed points since $dx/dN \neq 0$ and $dr/dN \neq 0$ during the course of evolution, but the differential equation (19) for $y(t)$ has a fixed point $y_f = 0$ as solution of $dy/dN = 0$ as $t \rightarrow \infty$. Fig.2 gives the corresponding evolution of the fractional energy densities. Starting with the initial value $\Omega_{ri} \simeq 1/2$, the radiation component Ω_r has a simple evolution of monotonic decrease. In contrast, the matter component Ω_m , starting with $\simeq 1/2$, increases quickly and approaches to ~ 1 around a redshift $(1+z) \sim 174$. At $(1+z) \sim 2.7$, Ω_m drops down and is dominated by Ω_y at $z \sim 0.35$. The YMC component Ω_y starts with the very small initial value and increases slowly and monotonically. Around $(1+z) \sim 2.7$, Ω_y has a quick increase, and around $z \sim 0.35$ it dominates over Ω_m . Observe in Fig.2 that the two curves of Ω_y for the two different initial values 10^{-10} and 10^{-2} , respectively, are almost overlapped into one curve. This pattern of degeneracy for Ω_y demonstrates vividly the fact that the cosmic evolution and the current status are insensitive to the initial condition of the YMC. As the result of evolution, at present ($z = 0$), one has $\Omega_y \sim 0.7$, $\Omega_m \sim 0.3$, and $\Omega_r \sim 10^{-5}$. Fig.3 shows the evolution of w_y and that of the effective EoS w_{eff} defined by

$$w_{eff} = \frac{p_m + p_y}{\rho_m + \rho_y}. \quad (31)$$

Both approach to -1 as $t \rightarrow \infty$, but, they can not across -1 . So there is no super-accelerating stage in the non-coupling model. Looking back Eq.(5) and (6), it is clear that in the non-coupling case the YM field strength will always stay above the critical value: $F \geq \kappa^2$. The asymptotic region at $t \rightarrow \infty$ corresponds to $F = \kappa^2$. This is a state with the dielectric constant $\epsilon = 0$. Thus, for the non-coupling case, no matter what kind of initial condition is given, the YMC always settles down to the state of $\epsilon = 0$ as a result of dynamic evolution. As for the current status $\Omega_y \sim 0.7$ and $\Omega_m \sim 0.3$, it has been achieved for the whole range of initial values in Eq.(29) at the fixed model parameter κ in Eq.(23). Therefore, the coincidence problem is solved in this model, but the fine-tuning problem still exists, i.e., we do not have an answer to the question why κ should have such a value as in Eq.(23). The non-coupling case with $Q_m = 0$ and $Q_r = 0$ has also been studied in [23, 24] with the initial condition being taken at a redshift $z \sim 10^{10}$ in the radiation dominated stage. There the evolution behavior found for the matter dominated era is similar to what is obtained here.

4. YMC Decaying into Matter

Consider the case that the YMC couples to the matter component only. In terms of the cosmic energy densities today, the radiation fraction is roughly $\Omega_r \sim 10^{-5}$, a relatively very small contribution, much lower than the other two components. Therefore, in this section we temporarily neglect its coupling with the YMC by setting $Q_r = 0$. There is only one free parameter Q_m since κ has been fixed. Then Eqs.(19), (20), and (21) reduce to

$$\frac{dy}{dN} + \frac{4y}{2+y} = -\frac{q_m}{Hh(2+y)e^y}, \quad (32)$$

$$\frac{dx}{dN} + 3x = \frac{q_m}{Hh}, \quad (33)$$

$$\frac{dr}{dN} + 4r = 0. \quad (34)$$

The dynamic evolution of the radiation is independent of the other two, and $\rho_r(t) \propto a^{-4}(t)$, as is seen in Eq.(34). To proceed further, one needs to know the coupling q_m to solve Eqs. (32) and (33). As mentioned earlier, in general there are two kinds of models depending on whether $Q_m > 0$ or $Q_m < 0$. In this section, we examine the model $Q_m > 0$ with the YMC decaying constantly into the matter. We call this the Model 1. It can be generically expressed as

$$Q_m = \Gamma \rho_y, \quad (35)$$

where $\Gamma > 0$ is of dimension $[time]^{-1}$ and measures the decay rate of the YMC energy density as well as the production rate of the matter energy density. Here Γ is simply taken as a model parameter describing phenomenologically the interactions between the YMC and the matter. In the following we discuss several cases for the parameter Γ . Substituting Eq.(35) into Eqs. (32) and (33) yields

$$\frac{dx}{dN} = \frac{\Gamma}{H} \frac{(1+y)e^y}{h} - 3x, \quad (36)$$

$$\frac{dy}{dN} = -\frac{\Gamma}{H} \frac{1+y}{(2+y)h} - \frac{4y}{2+y}. \quad (37)$$

1. Consider the simple case of a constant rate with

$$\Gamma/H = 0.5. \quad (38)$$

We have taken this magnitude of the rate Γ , so that the present status of the universe will be $\Omega_y \simeq 0.7$ and $\Omega_m \simeq 0.3$ as the outcome from our computation. Interestingly, in order to achieve this status, as a model parameter, the decay rate of the YMC needs to be of the same order of magnitude as the expansion rate of the universe, i.e. $\Gamma \sim H$. Now at the time t_i with $z \simeq 3454$ the initial YMC energy density is taken to be in the range

$$\Omega_{yi} = (10^{-10}, 3 \times 10^{-3}), \quad (39)$$

similar to Eq.(28), which corresponds to

$$y_i = (1, 15). \quad (40)$$

The initial values for the matter and the radiation components are given by

$$x_i = r_i = 1.0 \times 10^{10}, \quad (41)$$

which are a little bit smaller than that in Eq.(30). This is because in the case here the matter are being generated out of the decaying YMC during the course of evolution. Consequently, smaller initial values x_i and r_i are needed to arrive at the current status.

Given the different initial values of Ω_{yi} in Eq.(39), the corresponding initial values of $\Omega_{mi} = \Omega_{ri} = (1 - \Omega_{yi})/2$ also vary by a small amount. But for the different values of y_i in Eq.(40) we have taken the same set of values in Eq.(41) since the initial total energy density $\rho_{yi} + \rho_{mi} + \rho_{ri}$ itself at $z \simeq 3454$ has some errors. The results are given as functions of the redshift z in Fig.4, Fig.5, and Fig.6. The evolution of $\rho_y(t)$ is also similar to the non-coupling case. During the early stage $\rho_y(t)$ is lower than, and keeps track of $\rho_m(t)$. Later $\rho_y(t)$ levels off and approaches to a constant, and around $z \sim 0.48$ it starts to dominate over $\rho_m(t)$.

and the accelerating stage begins. In fact in Fig.4 for three different y_i there are three curves of the matter $\rho_m(t)$, respectively. However, these three curves are too close to each other so that they are overlapped. The similar is for $\rho_r(t)$. Note that, in the presence of coupling Q_m , the evolution of $\rho_m(t)$ is different from the non-coupling case, in that at the late stage around $z \sim 0$ it also levels off just like $\rho_y(t)$. In this sense, the evolution of the matter component is sort of bound to the YMC. In fact, as $t \rightarrow \infty$, the set of Eqs.(36) and (37) have the asymptotic behavior

$$\frac{dx}{dN} \rightarrow 0, \quad \frac{dy}{dN} \rightarrow 0, \quad (42)$$

and both $\rho_y(t)$ and $\rho_m(t)$ have asymptotic values. This will be addressed later in section 6.

As a novel feature of the coupling model, in contrast to the non-coupling case, now the EoS of the YMC as a function of time t , $w_y(t)$ crosses over -1 around $z \sim 2$, takes a value $w_y \simeq -1.1$ at present $z = 0$, and approaches $w_y \simeq -1.17$ asymptotically, as shown in Fig.6. This occurrence of crossing over -1 in this model can be understood, since the coupling makes the YMC to loss energy into the matter, consequently, the YM field strength F will drop down below the critical value κ^2 , leading to $\epsilon = by < 0$ and $w_y < -1$ as in Eq.(5). This can also be arrived by looking at the asymptotic region determined by the equation $\frac{dy}{dN} = 0$, which by Eq.(37) is just

$$\frac{\Gamma}{H} \frac{1+y}{h} + 4y = 0. \quad (43)$$

Recall that for the non-coupling $\Gamma = 0$ the asymptotic value is $y_f = 0$, yielding $w_y = -1$ by Eq.(5). Once $\Gamma > 0$, Eq.(43) yields an asymptotic value $y_f < 0$ as the solution, hence $w_y < -1$. Thus, when transferring energy to the matter, the YMC will eventually settle down in the state of $w_y < -1$, which is equivalent to a negative dielectric $\epsilon < 0$. Recently, there are some observational indications that the current value of EoS of dark energy w is less than -1 , for instance, $w = -1.023 \pm 0.090(stat) \pm 0.054(sys)$ from the 71 high redshift supernovae discovered during the first year SNLS [25], and $w = -1.21^{+0.15}_{-0.12}$ from the blind analysis of 21 high redshift supernovae by CMAGIC technique [26]. Of course, this is still to be observationally examined with higher confidence level in future. However, the crossing over -1 would be difficult for scalar models, except for quintom models at a price of introducing two scalar fields and an artificially designed potential [27]. As we just have demonstrated, in the YMC dark energy model with coupling $Q_m > 0$, this crossing is realized naturally. In general, the asymptotic value of w_y at $t \rightarrow \infty$ and the current value w_y at $z = 0$ as well, are determined by the asymptotic value of y through Eq.(5), and the latter is obtained from the combination of Eqs.(37) and (42), and thus depends on the ratio Γ/H . Thus the asymptotic value of w_y is determined by the ratio Γ/H of the two parameters of our model. For instance, the observed Eos of dark energy $w = -1.023$ from SNLS [25] can be obtained, in our model, by taking a little smaller decay rate $\Gamma/H = 0.13$ and slightly higher initial densities $x_i = r_i = 1.5 \times 10^{10}$, and the value $w = -1.21$ from the analysis by CMAGIC [26] can be obtained by taking a bit larger decay rate $\Gamma/H = 0.81$ and a slightly lower densities $x_i = r_i = 0.7 \times 10^{10}$, respectively. Fig.6 also shows that the effective w_{eff} can not cross -1 yet, and its asymptotic value is ~ -0.96 . Interestingly, this model predicts that in the upcoming future the dark energy density ρ_y will remain a constant slightly larger than today, and that the matter energy density ρ_m will be a constant slightly lower than today. Eventually the universe will settle down to a steady state with $\Omega_y \sim 0.85$, $\Omega_m \sim 0.15$, and $\Omega_r \sim 0$.

Therefore, this model yields a picture of evolutionary cosmos, the early part of which can account for the the past history of the expanding universe, i.e., that of the standard Big-Bang model, and the late part of which, i.e. the future of the universe, is similar to the Steady State Model [28] [29]. As a plus for the YMC model, there are no Big Rip singularities in finite time, since all the quantities ρ_y , p_y , and w_y are smooth function of the time t .

Although in the above we have only presented the results for the initial values within the range $\Omega_{yi} > 10^{-10}$ given in Eq.(28), as a matter of fact, we have worked out for even lower values $\Omega_{yi} < 10^{-10}$. For instance, we have calculated the case of $\Omega_{yi} \simeq 2 \times 10^{-12}$, i.e., $y_i = -0.9$. the initial energy density ρ_{yi} are even lower than the current values $\rho_y \simeq 0.7\rho_c$. The details are given by the curves denoted by $y_i = -0.9$ in Fig.4, Fig.5, and Fig.6. The evolution is such that $\rho_y(t)$ starts initially from the given very low value, increases (instead of decreasing) very quickly with time t , and approaches its corresponding asymptotic value $\simeq 0.7\rho_c$, as is seen in Fig.4. The evolutions for $\rho_m(t)$ and $\rho_r(t)$ are similar to the cases in Eq.(39). Thus, except the initial increasing of $\rho_y(t)$, the current status of the universe is the same as those in the range of Eq.(39).

2. Now consider a case of the YMC decaying into fermion pairs. As is known in QED, a constant electric field is unstable against decay into pair of particles [30]. Analogously, a constant ‘electric’ $SU(3)$ Yang-Mills field configuration of QCD is unstable against decay into quark pairs and gluon pairs [31] [32] [33]. Calculations have shown that for a gauge group $SU(N)$, due to the decay into fermions pairs and gauge boson pairs, the average local color electric field decreases at a rate

$$\frac{\partial E}{\partial t} = -cE^{3/2}, \quad (44)$$

where c is a dimensionless constant [32] [33] [20]. Notice that this rate is derived from the energy-momentum conversation of pair creation. From the point of view of particle physics, the $SU(2)$ YM field in our case should be allowed to have some fundamental interactions with other microscopic particles. At the moment we do not intend to proceed further to build up the detail of its interaction. Instead, we simply assume that, similar to the $SU(3)$ gauge field in QCD, the YM condensate, here a constant in space, also has the property of being unstable against decay into other particles, and the decay rate of the field strength has the same form as Eq.(44). Consequently, this in turn will give rise to the decay rate of the YMC energy density

$$\frac{\partial \rho_y}{\partial t} = -\Gamma \rho_y. \quad (45)$$

Making use of the chain rule relation $\frac{\partial \rho_y}{\partial t} = \frac{\partial \rho_y}{\partial E} \frac{\partial E}{\partial t}$ and Eq.(44), one obtains the following expression for the decay rate [20]

$$\Gamma = 2c\kappa^{\frac{1}{2}} \frac{2+y}{1+y} e^{\frac{y}{4}}. \quad (46)$$

depending on the coefficient constant c , which is treated as the model parameter in place of Γ . We choose the value of $c = 0.125H/\kappa^{1/2}$, so that

$$\frac{\Gamma}{H} = 0.25 \frac{2+y}{1+y} e^{\frac{y}{4}}. \quad (47)$$

As our computations show, the value of variable y at present stage is very small $y \sim 0$, so Eq.(47) yields $\Gamma \sim 0.5H$, quite close to that in Eq.(38). The initial condition is taken to be the same as Eqs.(40) and (41). Figs 7, 8, and 9 show the results, which are very similar to that in the previous case of the constant rate.

From Eq (13) it is seen that in the late-time asymptotic region when $\dot{\rho}_m \sim 0$ the matter generation rate is

estimated as $Q_m \simeq 3H\rho_m \simeq 10^{-46} g cm^{-3} s^{-1}$, a very small rate, equivalent to generation of ~ 0.3 protons in a cubic kilometer per year. This value is approximately equal to that in the Steady State Model [28] [29]. Thus in our model the particle pairs are continuously generated, at a very low rate, out of the vacuum filled with the YMC.

As is known, in order to continuously generate the cosmic matter, the Steady State Model has to introduce some C -field with negative energy [29], which is problematic in a physical theory. Here in our model, it is the effective quantum YMC plays the role of a matter-generator, there is no negative energy to occur in the proper range. The YMC has a positive energy and a negative pressure.

From Eq.(38) and (47) it seems that the overall behavior of the dynamic evolution is not sensitive to the particular form of the coupling Γ , as long as its magnitude is $\Gamma \sim 0.5H$. This has been confirmed in our examinations. For example, we have also investigated the case with the decay rate of the form

$$\Gamma/H = 0.5e^{-y}, \quad (48)$$

and the results are very similar to that in the previous case. Therefore, in Model 1 of the YM-matter coupling with the coupling $Q_m = \Gamma\rho_y$, as long as Γ is constant or depends on ρ_y , the overall features of the dynamic evolution are similar. We have also studied the cases with Γ depending on the matter ρ_m . It is found that, if the decay rate Γ depends on the matter ρ_m , for instance

$$\Gamma/H = bx, \quad (49)$$

where b is some constant and x is defined in Eq.(15), then for $b \leq 10^{-5}$ the evolution will be similar to the non-coupling case. When the constant $b \geq 10^{-3}$, the decay rate is too fast, and at start $\rho_y(t)$ drops down quickly, later it increases to its asymptotic value from below. To keep the paper short, we do not demonstrate these detailed graphs of the cases of Eqs.(48) and (49).

5. Matter Decaying into YMC

In section we study the situation in which the matter decaying constantly into the YMC with $Q_m < 0$, just opposite to the Model 1. We call this the Model 2. It can be generically expressed as

$$Q_m = -\Gamma\rho_m, \quad (50)$$

i.e. the matter transports energy into the YM condensate. Then Eqs. (32) and (33) reduce to

$$\frac{dx}{dN} + 3x = -\frac{\Gamma}{H} \frac{x}{h}, \quad (51)$$

$$\frac{dy}{dN} + \frac{4y}{2+y} = \frac{\Gamma}{H} \frac{x}{(2+y)e^y h}. \quad (52)$$

1. Consider the simple case of a constant decay rate

$$\Gamma/H = 0.02. \quad (53)$$

Again, here the value 0.02 has been taken, so that the resulting energy densities from our computation will be $\Omega_y \sim 0.7$ and $\Omega_m \sim 0.3$ at present. So in the Model 2 the decay rate of the matter into the YM condensate

needs to be almost two order of magnitude smaller than the expansion rate. The initial values of x_i are taken to be

$$x_i = r_i \simeq 1.8 \times 10^{10}. \quad (54)$$

Since the matter is decaying and constantly being converted into the YMC component, a slightly larger initial value of the matter has been taken than that in Eq.(41). The initial value of the YMC is taken to be $y_i = (1, 15)$, the same as Eq.(40). The results are given in Figs.10, 11, 12. Now w_y does not cross over -1 . It is interesting to find out that the evolution of the YM condensate behaves differently for two different ranges of y_i . For the higher range

$$y_i = (5, 15), \quad (55)$$

corresponding to $\Omega_{yi} = (5 \times 10^{-8}, 3 \times 10^{-3})$, all the quantities have similar evolution as the non-coupling case in Figs. 1, 2, 3. For the lower range

$$y_i = (1, 5), \quad (56)$$

however, the YMC has an instantly sudden increase during the initial stage, and quickly catches up the evolution pattern of the $y_i = 5$ case. This is in contrast to the smooth behavior on the higher range. Thus, in order to have a rather smooth evolution within the Model 2, the initial value y_i should be given by the higher range in Eq.(55). Moreover, we have also found that the overall behavior of the dynamic evolution is not sensitive to the particular form of the coupling Q_m . For instance, we have checked a case of

$$\Gamma/H = 0.02e^{-x}, \quad (57)$$

and the resulting evolutions are similar to the case of Eq.(53). Thus, the coincidence problem can also be solved in Model 2.

So far in Model 1 and Model 2, in regards to the coupling between the YMC and the the matter, we have not explicitly distinguished the baryons and the dark matter, and have assumed, for simplicity, the same coupling Q_m for both the baryons and the dark matter. We can roughly estimate the current value of the cross section corresponding to the collisions involving the baryons as in Eqs.(38), (47), (48), (49) with $b \leq 10^{-5}$, (53), and (57). For instance, take the baryon decay rate $\Gamma \sim bxH$ as in Eq.(49). Then, by definition, the rate is $\Gamma \sim vn\sigma$, where v is the baryon velocity, the baryon number density is $n = \rho_b/m_b \sim 0.04\rho_c/m_b$, and σ is the crossing section for the collisions between the baryons and the YM gauge bosons for this type of interaction . Then we can get an estimate:

$$\sigma \sim 25bm_bH/v\rho_c. \quad (58)$$

Taking $b \sim 10^{-5}$, $v \sim 10^3\text{km/s}$, the baryon mass $m_b \sim 0.94 \text{ Gev}$, the current Hubble constant for H , one has $\sigma \sim 6 \times 10^{-26} \text{ cm}^2$. This is an order lower than the Thomson's cross section $\sigma_T \simeq 6.7 \times 10^{-25} \text{ cm}^2$ for in QED. Similarly, letting $\Gamma \sim 0.5H$ as in Eqs.(38) and (47) for the YMC decaying into baryons, we would get $\sigma \sim 20 \times 10^{-25} \text{ cm}^2$ analogously, slightly greater than σ_T . Therefore, given this magnitude for the cross section σ in both cases, we would, in principle, be able to observe this kind of interactions occurring, either with the baryon being decaying into the YM boson pairs or the baryon pairs jumping out of the vacuum. However, as said earlier, the rate Γ for this kind of events is too low, giving $Q_m \sim 0.3$ proton generated in

one cubic kilometer per year. For the Galaxy of a volume $\sim 10^3(\text{kpc})^3$, this rate is roughly equivalent to an amount of mass $\sim 10^{-6}M_\odot$ generated per year, a small production rate. The chance may be small for directly detecting the event. Even if future experiments rule out or restrict the coupling with the baryons, one has to drop it or reduce its magnitude as a model parameter. Nevertheless, the coupling with the dark matter probably still remains. This is because the dark matter is usually assumed not to have interactions with ordinary particles, such as baryons, photons, etc. So it is difficult to directly detect productions of dark particle pairs and decays of dark particles.

6. Coupling with Both Matter and Radiation

It is quite natural to allow the YMC to couple with both the matter and the radiation simultaneously. Now we study Model 3 that the YMC decays into the matter and the radiation as well:

$$Q_m = \Gamma\rho_y > 0, \quad Q_r = \Gamma'\rho_y > 0. \quad (59)$$

Then Eqs. (19) – (21) reduce to

$$\frac{dy}{dN} = -\frac{\Gamma + \Gamma'}{H} \frac{1+y}{(2+y)h} - \frac{4y}{2+y}, \quad (60)$$

$$\frac{dx}{dN} = \frac{\Gamma}{H} \frac{(1+y)e^y}{h} - 3x, \quad (61)$$

$$\frac{dr}{dN} = \frac{\Gamma'}{H} \frac{(1+y)e^y}{h} - 4r. \quad (62)$$

Consider the case of the constant decay rates

$$\Gamma/H = 0.5, \quad \Gamma'/H = 1.8 \times 10^{-4}. \quad (63)$$

Note that Γ' is lower than Γ by three orders of magnitude. These values of coupling are taken so that the current values are $\Omega_y \simeq 0.7$, $\Omega_m \simeq 0.3$, $\Omega_\gamma \simeq 8.6 \times 10^{-5}$ (including massless neutrinos). The initial condition is the same as in Eqs.(40) and (41). The results are given in Figs.13, 14, 15. As before, the particular form of the couplings is not important for the overall behavior of evolution. For instance, we have also examined the case

$$\Gamma/H = 0.25 \frac{2+y}{1+y} e^{\frac{y}{4}}, \quad \Gamma'/H = 0.9 \times 10^{-4} \frac{2+y}{1+y} e^{\frac{y}{4}}, \quad (64)$$

based on an analogous consideration to Eq.(47). The evolution is similar to the case of Eq.(63). In these two cases of Model 3, due to the couplings with the YMC, both the energy densities, ρ_m and ρ_r , level off around $z \sim 0$, and w_y crosses over -1 around $z \simeq 2.5$. Thus all the three components of cosmic energy will remain constant in future, and the state of the universe will keep almost as it is today. This is similar to the Steady State universe [28] [29]. Hence, according to Model 3, the past history of the universe is consistent with the conventional standard Big Bang model, and from now on, the cosmic evolution tends to that of a steady state, that is, the universe will remain similar to it is today, but with $\Omega_y \sim 0.85$, $\Omega_m \sim 0.15$, and $\Omega_r \sim 10^{-5}$.

We have also studied the case with both the matter and radiation decaying into the YMC. We call this Model 4. The couplings are such that

$$Q_m = -\Gamma\rho_m < 0, \quad Q_r = -\Gamma'\rho_r < 0. \quad (65)$$

We take

$$\Gamma/H = 0.02, \quad \Gamma'/H = 1.8 \times 10^{-4}. \quad (66)$$

The initial condition for the YMC is the same as Eqs.(55) and (56) and the initial densities for the matter and radiation are the same as in Eq.(54). The resulting evolution is qualitatively similar to those in the Model 2 with $Q_m < 0$. Approximately, $\rho_m(t) \propto a(t)^{-3}$, $\rho_r(t) \propto a(t)^{-4}$, and $\rho_y(t)$ also has a scaling solution and exits the scaling regime, just like in the non-coupling case. The EoS of the YMC w_y approaches to -1 from above, but does not cross over -1 . To keep the paper short and concise, we will not repeat these details and not give the corresponding graphs here anymore.

7. Asymptotic Behavior and Stable Attractor

It is interesting to investigate the asymptotic behavior of the dynamical evolution. First we study the Model 1 with the YMC coupling to the matter component only. Now since the evolution of the the radiation component is independent of the the YMC and the matter, and the value of r at late time is much less than the other variables, so it can be neglected in the analysis of the fixed point. To find the fixed points, one sets $dx/dN = dy/dN = 0$ in Eqs. (32) and (33), and obtains the relations at the fixed point:

$$x_f = -\frac{4}{3}y_f e^{y_f}, \quad (67)$$

$$\frac{\Gamma}{H}\left(1 + \frac{1}{y_f}\right) = -4\sqrt{1 - \frac{y_f}{3}}e^{y_f/2}, \quad (68)$$

where the sub-index f refers to the the respective values at the fixed point. From these two equations one can write the asymptotic value of the fractional matter density

$$\Omega_{mf} = \frac{4y_f}{y_f - 3}. \quad (69)$$

Eqs. (67) and (68) depend on the value of the ratio Γ/H , so does the solution (x_f, y_f) . Here we consider the case that Γ/H is constant. Because Ω_{mf} must be larger than 0 and smaller than 1, so y_f must be in the range from -1 to 0 at the fixed point. Thus, as long as $\Gamma/H \in (0, \infty)$ there will exist fixed points. To be specific, consider $\Gamma/H = 0.5$. In the region of physics there is only one fixed point:

$$(x_f, y_f) = (0.13666, -0.11499). \quad (70)$$

This is, in terms of the respective densities,

$$(\rho_{mf}, \rho_{yf}) = \frac{1}{2}b\kappa^2(0.13666, 0.78888). \quad (71)$$

The stability of this fixed point can be analyzed in the conventional way as follows. Because the two equations for x and y are nonlinear, a local analysis can be given by linearizing the two evolution Eqs. (36) and (37). By a standard procedure, expanding $x = x_f + \varepsilon$ and $y = y_f + \eta$, where ε and η are small perturbations around the fixed point, and keeping up to the first order of small perturbations, Eqs. (36) and (37) reduce to

$$\frac{d}{dN} \begin{pmatrix} \varepsilon \\ \eta \end{pmatrix} = M \begin{pmatrix} \varepsilon \\ \eta \end{pmatrix}, \quad (72)$$

where M is a 2×2 matrix depending on the values of x_f , y_f , and Γ/H , whose the elements are

$$\begin{aligned} M_{11} &= - \left[\frac{\Gamma}{H} \frac{(1+y_f)e^{y_f}}{2h_f^3} + 3 \right], \\ M_{12} &= \frac{\Gamma}{H} \frac{e^{y_f}}{h_f} \left\{ (1+y_f) \left[1 - \frac{(2+y_f)e^{y_f}}{2h_f^2} \right] + 1 \right\}, \\ M_{21} &= \frac{\Gamma}{H} \frac{1+y_f}{2(2+y_f)h_f^3}, \\ M_{22} &= \frac{\Gamma}{H} \left[\frac{(1+y_f)e^{y_f}}{2h_f^3} - \frac{1}{(2+y_f)^2 h_f} \right] - \frac{8}{(2+y_f)^2}, \end{aligned}$$

where $h_f = \sqrt{(1+y_f)e^{y_f} + x_f + r_f}$. The general solution for the linear perturbations is of the form

$$\varepsilon = C_1 e^{\mu_1 N} + C_2 e^{\mu_2 N} \quad (73)$$

$$\eta = C_3 e^{\mu_1 N} + C_4 e^{\mu_2 N} \quad (74)$$

where μ_1 and μ_2 are eigenvalues of M . If they are both negative, the fixed point (x_f, y_f) is stable, and the solution is called an attractor. For the case of $\Gamma/H = 0.5$ one finds the matrix

$$M = \begin{pmatrix} -3.22146 & 0.50112 \\ 0.13180 & -2.17625 \end{pmatrix},$$

and its two eigenvalues $\mu_1 = -3.28123$ and $\mu_2 = -2.11648$, respectively, both negative. Thus the fixed point of this model is stable, and is an attractor. For illustration, we plot in Fig.16 the phase graph of trajectories, each trajectory starts with a different initial condition, and ends up at the fixed point of Eq.(70). As a matter of fact, one can check that the asymptotic behavior of other cases of Model 1 are also stable fixed points.

The analysis of the asymptotic behavior can be done analogously for Model 3 with the couplings to the matter and the radiation. Setting $dx/dN = dy/dN = dr/dN = 0$ in Eqs. (60), (61) and (62) one has the following three relations at the fixed point:

$$4y_f = - \frac{\Gamma + \Gamma'}{H} \frac{1+y_f}{h_f}, \quad (75)$$

$$3x_f = \frac{\Gamma}{H} \frac{(1+y_f)e^{y_f}}{h_f}, \quad (76)$$

$$4r_f = \frac{\Gamma'}{H} \frac{(1+y_f)e^{y_f}}{h_f}. \quad (77)$$

The fixed point (x_f, y_f, r_f) as the solution of this set of equations depends on the ratios of rates Γ/H and Γ'/H as well. Consider the constant Γ and Γ' . The local analysis of the stability of the fixed point can be carried out similarly. By setting $x = x_f + \varepsilon$, $y = y_f + \eta$, and $r = r_f + \gamma$, where ε , η , and γ are small perturbations around the fixed point, one has the equations

$$\frac{d}{dN} \begin{pmatrix} \varepsilon \\ \eta \\ \gamma \end{pmatrix} = M' \begin{pmatrix} \varepsilon \\ \eta \\ \gamma \end{pmatrix}, \quad (78)$$

where M' is a 3×3 matrix depending on the values of x_f , y_f , r_f , Γ/H and Γ'/H , whose the elements are:

$$M'_{11} = - \left[\frac{\Gamma}{H} \frac{(1+y_f)e^{y_f}}{2h_f^3} + 3 \right],$$

$$\begin{aligned}
M'_{12} &= \frac{\Gamma}{H} \frac{e^{y_f}}{h_f} \left\{ (1 + y_f) \left[1 - \frac{(2 + y_f)e^{y_f}}{2h_f^2} \right] + 1 \right\}, \\
M'_{13} &= -\frac{\Gamma}{H} \frac{(1 + y_f)e^{y_f}}{2h_f^3}, \\
M'_{21} &= \frac{\Gamma + \Gamma'}{H} \frac{1 + y_f}{2(2 + y_f)h_f^3}, \\
M'_{22} &= \frac{\Gamma + \Gamma'}{H} \left[\frac{(1 + y_f)e^{y_f}}{2h_f^3} - \frac{1}{(2 + y_f)^2 h_f} \right] - \frac{8}{(2 + y_f)^2}, \\
M'_{23} &= \frac{\Gamma + \Gamma'}{H} \frac{1 + y_f}{2(2 + y_f)h_f^3}, \\
M'_{31} &= -\frac{\Gamma'}{H} \frac{(1 + y_f)e^{y_f}}{2h_f^3}, \\
M'_{32} &= -\frac{\Gamma'}{H} \frac{(2 + y_f)e^{y_f}}{h_f} \left[\frac{(1 + y_f)e^{y_f}}{2h_f^2} + 1 \right] \\
M'_{33} &= -\left[\frac{\Gamma'}{H} \frac{(1 + y_f)e^{y_f}}{2h_f^3} + 4 \right].
\end{aligned}$$

Consider the specific case $\Gamma/H = 0.5$ and $\Gamma'/H = 1.8 \times 10^{-4}$. Substituting these into Eqs.(75), (76) and (77) yields the unique fixed point given by

$$(x_f, y_f, r_f) = (0.13666, -0.11503, 4 \times 10^{-5}), \quad (79)$$

and the matrix

$$M' = \begin{pmatrix} -3.22149 & 0.50110 & -0.22149 \\ 0.13187 & -2.17631 & 0.13187 \\ -0.00008 & 0.00045 & -4.00008 \end{pmatrix}.$$

The three eigenvalues of the matrix M' are found to be -4.00015 , -3.28124 and -2.11649 , each being negative. Therefore, this attractor of the Model 3 is also stable. Notice that there are three quantities (x, y, r) in the Model 3, so we need the two phase graphs of trajectories. They are plotted in Fig.17 for (x, y) and Fig.18 for (r, y) , separately. Each trajectory starts with a different initial condition, and ends up at the fixed point of Eq.(79). One can check the dynamics of other cases in Model 3 also have a stable attractor.

As for Model 2 and Model 4, only $\rho_y(t)$ has an asymptotic constant value and has a stable attractor.

8. Conclusion and Discussion.

Our motivation of this study is to investigate the coincidence problem for the cosmic dark energy in a spatially flat universe. We have presented a detailed and comprehensive analysis of the model of the effective YMC dark energy interacting with the matter and radiation. This work has been an extended development of our previous work on the non-coupling YMC dark energy model. Through the Friedmann equation and the dynamic equations for each cosmic component, once the couplings between these components are specified, the overall cosmic evolution is fully determined by the initial conditions of these three components. We have studied the evolution for the matter-dominated era starting from the equality of radiation-matter at $z \sim 3454$

The major results of this work are the following.

Given the initial dominant matter and radiation $\Omega_{mi} = \Omega_{yi} \simeq 1/2$ and the subdominant YMC energy density $\Omega_{yi} \leq 10^{-2}$, no matter what kind of coupling between the YMC and the matter, or between the YMC and the radiation, the evolution is such that the YMC is subdominant to, and keeps track of, the matter, until later, at a redshift $z \sim 0.48$ for a coupling $Q_m > 0$, or $z \sim 0.35$ for a coupling $Q_m < 0$, the YMC becomes dominant over the matter. The era is followed by a subsequent accelerating era driven by the dominant YM dark energy. As the evolution outcome, the universe arrives at the present state with $\Omega_y \sim 0.7$, $\Omega_r \sim 0.3$, and $\Omega_m \sim 10^{-5}$. It is very important to note that this has been achieved for a variety of coupling forms Q_m and Q_r , and, nevertheless, under a very broad range of initial condition $\Omega_{yi} \simeq (10^{-10}, 3 \times 10^{-3})$.

If the YMC decays only into the matter, as a result of the coupling in Model 1, to achieve the present state of the universe, the decay rate needs to be of the same order of magnitude of the expansion rate of the universe, i.e., $\Gamma \sim 0.5H$. Moreover, for this coupled system, as $t \rightarrow \infty$, both ρ_y and ρ_m asymptotically approach to constants, respectively. That is, for the system there is a unique attractor. Furthermore, as our analysis has shown, this attractor is stable. As an interesting behavior, the EoS for the YMC w_y always crosses over -1 around $z \simeq 2.5$, and the present value is $w_y \sim -1.1$. This crossing -1 seems to be favored by the recent preliminary observations on SN Ia.

When the YMC decays into both the matter and the radiation as in Model 3, there are two parameters $\Gamma \sim 0.5H$ and $\Gamma' \sim 1.8 \times 10^{-4}H$, representing the respective decay rate. The evolutionary behavior is almost the same as Model 1, and w_y crosses over -1 . Moreover, the radiation energy density also asymptotically approaches to a constant, as $t \rightarrow \infty$, like the YMC and the matter components. Most of the conclusions are the same as of Model 1.

On the other hand, if the matter decays into the YMC as in Model 2 with a rate $\Gamma \sim 0.02H$, or if both the matter and radiation decay into the YMC as in Model 4 with rates $\Gamma \sim 0.02H$ and $\Gamma' \sim 1.8 \times 10^{-4}H$, respectively, then only ρ_y asymptotically approaches to a constant. w_y approaches to -1 , but does not cross over -1 . The evolution is nearly similar to the non-coupling case.

Therefore, for all four types of models that we have studied, the coincidence problem can be naturally solved by introducing the effective YMC as the dark energy at the fixed parameter κ given in Eq.(38). The present state of the universe is a natural result of the dynamic evolution. The past history of the evolving universe is that of the standard Big Bang model, and the future of the universe depends on the details of the coupling. If there is no coupling, or if the matter decays, or both matter and radiation decay, into the YMC, as in Model 2 and in Model 4, the matter and the radiation will keep on decreasing as $\rho_m(t) \propto a(t)^{-3}$ and $\rho_r(t) \propto a(t)^{-4}$. If the YMC decays into the matter only as in Model 1, then $\rho_m(t)$ will asymptotically remain as constant, like $\rho_y(t)$ does, but $\rho_r(t) \propto a(t)^{-4}$. If the YMC decays into both the matter and radiation as in Model 3, then all the components $\rho_y(t)$, $\rho_m(t)$, and $\rho_r(t)$ will asymptotically remain as constant. In Model 1 and Model 3 the future of the universe is a steady state, quite similar to that of the Steady State model, thus, in a sense, these two models bridge between the Big Bang model and the Steady State model.

The distinguished characteristics of the YMC dark energy model are the following.

The YM field is known to be indispensable to particle physics, the effective YMC employed in our work comes from quantum corrections up to 1-loop . Therefore, there is no room to adjust the form of the

effective Lagrangian. This is in contrast to scalar field models, which have to design the form of potential and sometimes even the form of kinetic energy.

The solution of coincidence problem has been relying on the parameter κ in all our models. Viewed from the standard model of particle physics, the energy scale by κ is much smaller than the other known microscopic energy scales. And this stands as the fine-tuning problem for any current cosmological model so far, and for our model as well. However, if the YM field in our model is regarded as a fundamental gauge field with κ being the energy scale for this new physics, then the fine-tuning problem is traced up to the new physics. When the couplings are included, there are two more parameters Γ and Γ' . But the present state of the universe requires that Γ be roughly the same order of magnitude of the expansion rate $\Gamma \sim 0.5H$, and Γ' be roughly three order lower. Since $H \sim \sqrt{G\kappa^2} = \kappa/m_{pl}$ with m_{pl} being the Planck mass, thus the couplings $\Gamma \sim \kappa/m_{pl}$ and $\Gamma' \sim 10^{-3}\kappa/m_{pl}$ are also associated with the scale κ .

On the dynamic evolution, in comparison with scalar models, our models have the following features. All our models, for a broad range of the initial condition and for a variety of the coupling forms, automatically have the scaling property, i.e., $\rho_y(t)$ is initially subdominant to, and keeps track of to the matter. The accelerating stage begins only quite recently around a redshift $z \sim (0.35, 0.48)$. This will allow the Big Bang cosmology to remain without drastic modifications. Besides, all our models have only one stable fixed point, uniquely determined by the ratio Γ/H and has nothing to do with the initial conditions Ω_{yi} . Moreover, all the quantities in our model, especially $\rho_y(t)$ and $p_y(t)$ are continuous functions of t . So there is no Big Rip singularities in our models. Interestingly, as a function of t , w_y behaves quite smoothly during the evolution, going from $\sim 1/3$, approaching to -1 . And in the case of the YMC decaying, w_y crosses over -1 at $z \sim 2$, acquires the present value ~ -1.1 , and settles down to an asymptotic value ~ -1.17 .

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Figure 1: Non-coupling case: The evolution of energy densities. For a wide range of initial conditions $\rho_{yi} = (10^{-10}, 10^{-2})\rho_{mi}$, there always exists a scaling solution during the early stages, and $\rho_y(t)$ levels off and becomes dominant around $z \sim 0.35$.

Figure 2: Non-coupling case: The evolution of fraction of energy densities. The two curves of Ω_y for $y_i = 1$ and $y_i = 16.12$ are very close to each other, and nearly overlap in this figure.

Figure 3: Non-coupling case: The evolution of EoS. w_y approaches to -1 as $t \rightarrow \infty$, but does not cross over -1 .

Figure 4: Model 1 with $Q_m > 0$ for $\Gamma/H = 0.5$: The evolution of energy densities with the YMC decaying into the matter. For a wide range of initial conditions $\rho_{yi} = (10^{-10}, 10^{-2})\rho_{mi}$, i.e., $y_i = (1, 15)$, there always exists a scaling solution during the early stages, and $\rho_y(t)$ levels off and becomes dominant around $z \sim 0.48$. Due to the coupling, $\rho_m(t)$ also levels off at late time.

Figure 5: Model 1 with $Q_m > 0$ for $\Gamma/H = 0.5$: The evolution of fractional energy densities in the same model as in Fig.4.

Figure 6: Model 1 with $Q_m > 0$ for $\Gamma/H = 0.5$: The evolution of EoS in the same model as in Fig.4 and Fig.5. Due to the coupling, w_y crosses over -1 , takes on value ~ -1.1 at $z = 0$, and approaches -1.17 asymptotically. This is in contrast to the non-coupling case.

Figure 7: Model 1 with $Q_m > 0$ for $\Gamma/H = 0.25e^{\frac{y}{4}\frac{2+y}{1+y}}$: The evolution of energy densities with the YMC decaying into the matter. For a wide range of initial conditions $\rho_{yi} = (10^{-10}, 10^{-2})\rho_{mi}$, there always exists a scaling solution during the early stages, and $\rho_y(t)$ levels off and becomes dominant around $z \sim 0.48$. Due to the coupling, $\rho_m(t)$ also levels off at late time. This is quite similar to the case of a constant rate $\Gamma/H = 0.5$ in Fig.4.

Figure 8: . Model 1 with $Q_m > 0$ for $\Gamma/H = 0.25e^{\frac{y}{4}\frac{2+y}{1+y}}$: The evolution of fractional energy densities in the same model as in Fig.7.

Figure 9: Model 1 with $Q_m > 0$ for $\Gamma/H = 0.25e^{\frac{y}{4}\frac{2+y}{1+y}}$: The evolution of EoS in the same model as in Fig.7 and Fig.8. Again, due to the coupling, w_y crosses over -1 , and takes on value ~ -1.1 at $z = 0$.

Figure 10: Model 2 with $Q_m < 0$ for the constant coupling $\Gamma/H = 0.02$: The evolution of energy densities with the matter decaying into the YMC.

Figure 11: Model 2 with $Q_m < 0$ for the constant coupling $\Gamma/H = 0.02$: The evolution of fractional energy densities in the same model as in Fig.10.

Figure 12: Model 2 with $Q_m < 0$ for the constant coupling $\Gamma/H = 0.02$: The evolution of EoS in the same model as in Fig.10 and Fig.11. Since the YMC gets energy from the matter, w_y approaches to -1 , but does not cross over -1 .

Figure 13: Model 3 with $Q_m > 0$ and $Q_r > 0$ for $\Gamma/H = 0.5$ and $\Gamma'/H = 0.00018$: The evolution of energy densities with the YMC decaying into both the matter and the radiation. For a wide range of initial conditions $\rho_{yi} = (10^{-10}, 10^{-2})\rho_{mi}$, there always exists a scaling solution during the early stages, and $\rho_y(t)$ levels off and becomes dominant around $z \sim 0.48$. Note that, due to coupling, both $\rho_m(t)$ and $\rho_r(t)$ level off like $\rho_y(t)$.

Figure 14: The evolution of fraction of energy densities in the same model as in Fig.13.

Figure 15: Model 3 with $Q_m > 0$ and $Q_r > 0$ for $\Gamma/H = 0.5$ and $\Gamma'/H = 0.00018$: The evolution of EoS in the same model as in Fig.13 and Fig.14. Note that w_y also crosses over -1 and takes on a value ~ -1.1 at $z = 0$.

Figure 16: Model 1 with $Q_m > 0$ for $\Gamma/H = 0.5$: The trajectories in the phase plane. Each trajectory starts with a different initial condition. All of them approach the fixed point $(x_f, y_f) = (0.13666, -0.11499)$.

Figure 17: Model 3 with $Q_m > 0$ and $Q_r > 0$ for $\Gamma/H = 0.5$ and $\Gamma'/H = 0.00018$: The trajectories in the phase plane (x, y) . Each trajectory starts with a different initial condition. All of them approach the fixed point $(x_f, y_f, r_f) = (0.13666, -0.11503, 4 \times 10^{-5})$. The parameters are the same as in Fig.13.

Figure 18: Model 3 with $Q_m > 0$ and $Q_r > 0$ for $\Gamma/H = 0.5$ and $\Gamma'/H = 0.00018$: The trajectories in the phase plane (r, y) . The parameters are the same as in Fig.13.



































